Relay identification and control of anisochronic systems in RMS ring

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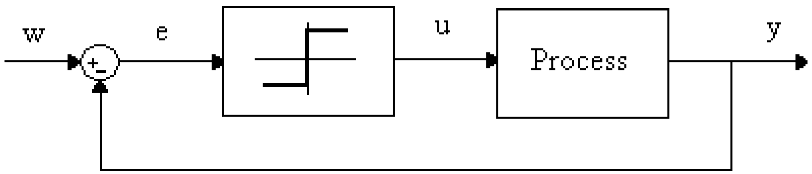
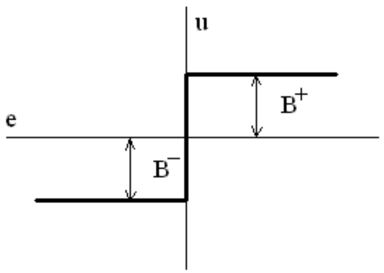
Relay feedback test

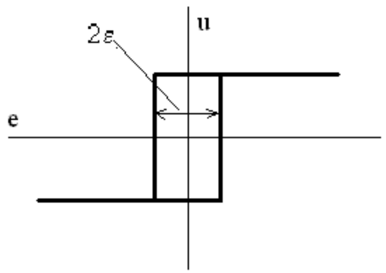
Relay feedback test



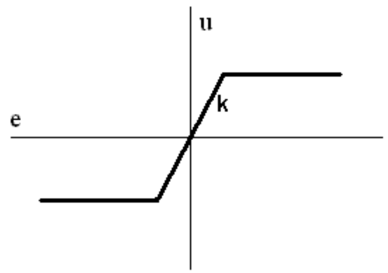
Types of relays (nonlinearities)



On-off (biased) relay



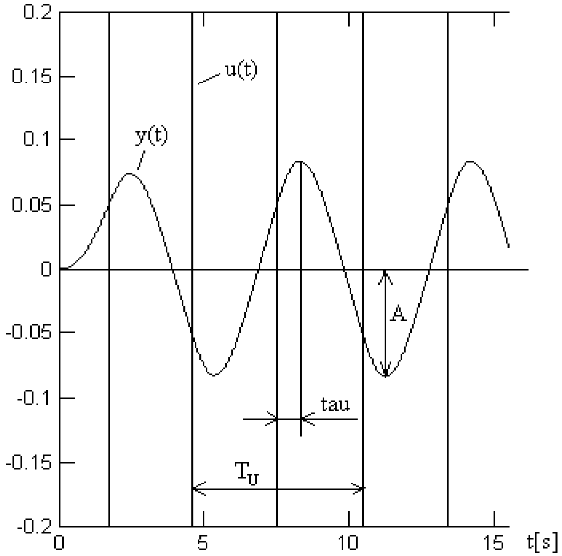
Relay with hysteresis



Saturation relay

Relay feedback test

Stable oscillations



$$\Rightarrow \omega_u,\ A$$

$$R(A)G(\mathrm{j}\omega_u) = -1 + 0\mathrm{j}$$

$$\left|R(A)G(\mathrm{j}\omega_u)\right| = 1$$

$$\arg\left[R(A)G(\mathrm{j}\omega_u)\right] = -\pi$$

On-off (biased) relay: $R(A) = \dfrac{4B}{\pi A}$

Relay with hysteresis:

$$R(A) = \frac{4B}{\pi A}\left[\sqrt{1-\left(\frac{\varepsilon}{A}\right)^2} - \mathrm{j}\frac{\varepsilon}{A}\right]$$

Relay feedback test

Conventional model

$$G(s) = \frac{K\exp(-\tau s)}{Ts+1} = \frac{b_0\exp(-\tau s)}{s+a_0}$$

Parameters estimation:

$$K = \frac{\int_0^{iT_u} y(t)dt}{\int_0^{iT_u} u(t)dt};\quad i = 1,2,3,\ldots \qquad T = \frac{T_u}{2\pi}\cdot\sqrt{\frac{16\cdot K^2\cdot B^2}{\pi^2\cdot A^2}-1}$$

$$\tau = \frac{T_u}{2\pi}\left[\pi - 2arctg\frac{2\pi T}{T_u} - arctg\frac{\varepsilon}{\sqrt{A^2-\varepsilon^2}}\right]$$

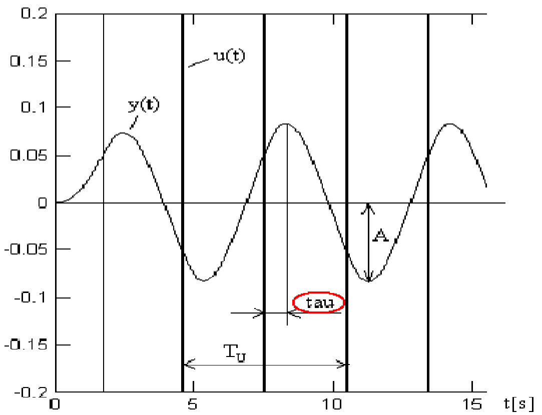
Relay feedback test

Anisochronic model

$$G(s)=\frac{Y(s)}{U(s)}=\frac{K\exp(-\tau s)}{Ts+\exp(-\vartheta s)}$$

Conventional parameters estimation:

$$K=\frac{\int_0^{iT_u} y(t)dt}{\int_0^{iT_u} u(t)dt};\quad i=1,2,3,\ldots$$



$$\frac{4KB}{\pi A}\frac{1}{\sqrt{(T\omega_u)^2-2T\omega_u\sin(\vartheta\omega_u)+1}}=1$$

$$\arctan\left[\frac{\sin(\vartheta\omega_u)-T\omega_u}{\cos(\vartheta\omega_u)}\right]-\tau\omega_u=-\pi$$

⇓

$$T=\frac{\sin(\vartheta\omega_u)-\tan(\tau\omega_u)\cos(\vartheta\omega_u)}{\omega_u}$$

$$\vartheta=\frac{1}{\omega_u}\arccos\left[\pi-\frac{4KB}{\pi A}\cos(\tau\omega_u)\right]$$

Relay feedback test

Alternative evaluation of limit cycles in time domain

Plant input and output:

$$u(t) = u_0 \sin(t\omega_u) = \frac{4B}{\pi}\sin(t\omega_u)$$

$$y(t) = y_0 \sin(t\omega_u) = -A\sin(t\omega_u)$$

Differential equation:

$$Ty_0\omega_u \cos(t\omega_u) + y_0 \sin[(t-\vartheta)\omega_0] = Ku_0 \sin[(t-\tau)\omega_0]$$

By selection of time value :

$$t = \omega_u^{-1}(2k\pi):\ Ty_0\omega_u - y_0 \sin(\vartheta\omega_u) + Ku_0 \sin(\tau\omega_u) = 0$$

$$t = \omega_u^{-1}(2k\pi + 0.5):\ -y_0 \cos(\vartheta\omega_u) + Ku_0 \cos(\tau\omega_u) = 0$$

=> The same solution as according to a convention method

Another interesting result:

$$\frac{\cos(\vartheta\omega_u)}{\cos(\tau\omega_u)} = \frac{Ku_0}{y_0} = -\frac{4KB}{\pi A}$$

Relay feedback test

Autotune variable technique (ATV)

1) Standard relay test
2) Additional delay => new ultimate values $\tilde{\omega}_u, \tilde{A}$

$$T\tilde{y}_0\tilde{\omega}_u \cos(\phi_D) + \tilde{y}_0 \sin(\phi_D - \vartheta\tilde{\omega}_u) + Ku_0 \sin(\tau\tilde{\omega}_u) = 0$$

$$-T\tilde{y}_0\tilde{\omega}_u \sin(\phi_D) + \tilde{y}_0 \cos(\phi_D - \vartheta\tilde{\omega}_u) - Ku_0 \cos(\tau\tilde{\omega}_u) = 0$$

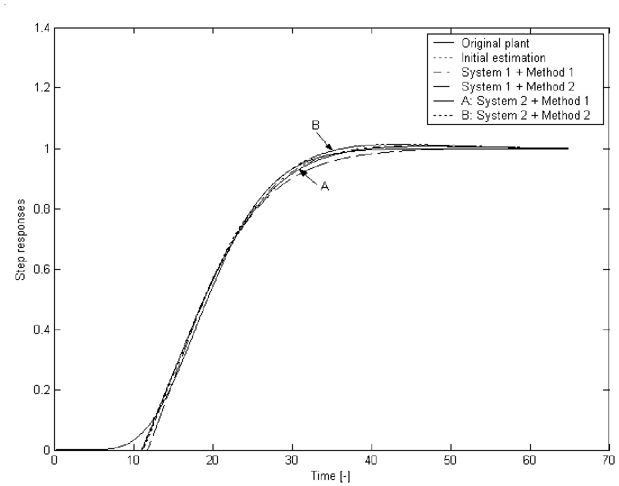
Plus: Parameter estimation using “primary” limit cycle information only
Minus: Set of nonlinear algebraic equations => numeric solution

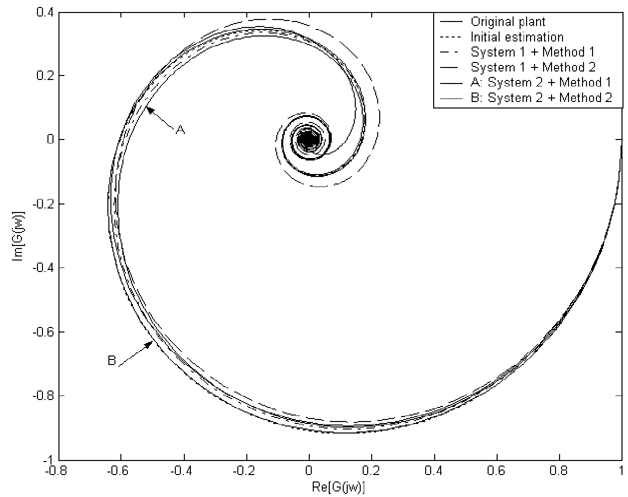
Relay feedback test

Example

$$G(s)=\frac{1}{(2s+1)^{10}}\Rightarrow \hat{G}(s)=\frac{K\exp(-\tau s)}{Ts+\exp(-\vartheta s)}$$

Parameter	Initial estimation	Σ_1 Method 1	Σ_1 Method 2	Σ_2 Method 1	Σ_2 Method 2
τ	11.21	11.14	11.82	11.05	11.26
T	15.3	15.34	14.04	15.6	15.18
ϑ	6.89	6.65	5.28	6.71	6.78





Relay feedback test

Criteria for step responses

$$J_{S1} = \sum_{i=1}^{N} \left[h_M(t_i) - h(t_i)\right]^2$$

$$J_{S2} = \sum_{i=1}^{N} t_i \left[h_M(t_i) - h(t_i)\right]^2$$

Criterion for Nyquist plots

$$J_F = \sum_{i=1}^{N} \left[\left(P_{M,i} - P_i\right)^2 + \left(Q_{M,i} - Q_i\right)^2 \right]$$

$$P_{M,i} = \Re\left[G_M(j\omega_i)\right],\ P_i = \Re\left[G(j\omega_i)\right],$$

$$Q_{M,i} = \Im\left[G_M(j\omega_i)\right],\ Q_i = \Im\left[G(j\omega_i)\right]$$

	J_F ($\Delta\omega_i = 10^{-3}$)	J_F (log. scale)	J_{S1}	J_{S2}
Initial estimation	60.94521	0.0602	0.2945	0.6343
Σ_1 Method 1	64.8645	0.0646	0.2628	0.4636
Σ_1 Method 2	85.5224	0.0883	0.4695	0.9884
Σ_2 Method 1	64.5455	0.0654	0.2646	0.4839
Σ_2 Method 2	61.9589	0.0609	0.2856	0.5803

Algebraic control in R_{MS} ring

Algebraic control in R_{MS} ring

- R_{MS} = ring of stable and proper retarded quasipolynomial (RQ) meromorphic functions
- RQ-meromorphic functions: description of a general term in R_{MS}

$$T(s) = \frac{y(s)}{x(s)} = \frac{y_0(s)\exp(-\tau s)}{x(s)}$$

where $y_0(s)$ is a (quasi)polynomial

$x(s)$ is a stable (quasi)polynomial

τ is non-negative

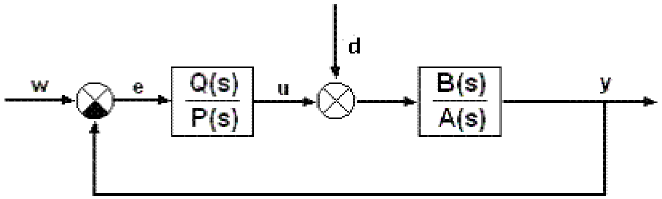
deg $x(s)$ is greater than or equal to deg $y(s)$ => Properness

- Transfer function is expressed as a ratio of two terms in R_{MS} – i.e. rational field over R_{MS}

Algebraic control in R_{MS} ring

Example 1: Conventional plant with time delay

$$G(s)=\frac{b_0\exp(-\tau s)}{s+a_0}=\frac{\dfrac{b_0\exp(-\tau s)}{s+m_0}}{\dfrac{s+a_0}{s+m_0}}=\frac{B(s)}{A(s)}$$



where $m>0$ is a scalar parameter

$B(s)$, $A(s)$ are terms over R_{MS}

Stabilization: Diophantine equation

$$AP+BQ=1\Leftrightarrow(s+a_0)P_0(s)+b_0Q_0(s)\exp(-\tau s)=s+m_0$$

with Youla-Kucera parameterization

$$P=P_0+BT,\qquad Q=Q_0-AT$$

$$Q=1\Rightarrow P(s)=\frac{s+m_0-b_0e^{-\tau s}}{s+a_0}\qquad\frac{Q(s)}{P(s)}=\frac{1+\dfrac{s+a_0}{s+m_0}T(s)}{\dfrac{s+m_0-b_0e^{-\tau s}}{s+a_0}-\dfrac{b_0e^{-\tau s}}{s+m_0}T}$$

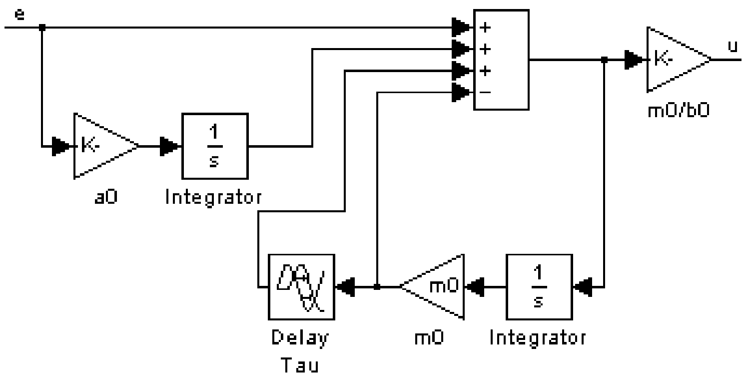
Algebraic control in R_{MS} ring

For both asymptotic tracking and disturbance rejection…

Stepwise reference and disturbance: $W(s)=\dfrac{H_w(s)}{F_w(s)}=D(s)=\dfrac{H_d}{F_d}=\dfrac{\frac{k}{s+m}}{\frac{s}{s+m}}$

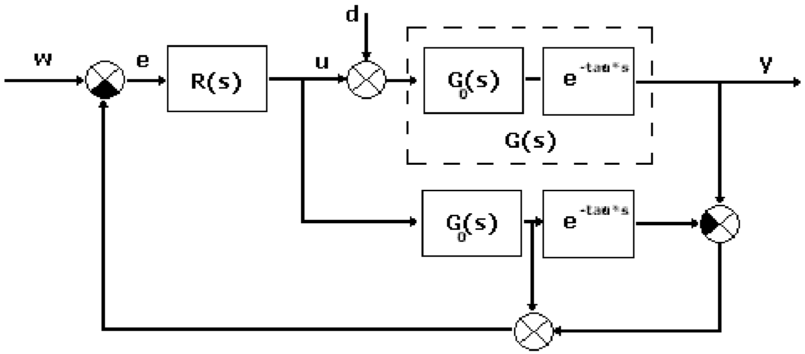
F_W and F_S divide P: $T(s)=\dfrac{\kappa(s+m_0)}{s+a_0};\ \kappa=\dfrac{m_0}{b_0}-1$

Final controller: $G_R(s)=\dfrac{m_0}{b_0}\dfrac{s+a_0}{s+m_0\left(1-e^{-\tau s}\right)}$



Algebraic control in R_{MS} ring

Comparison with Smith predictor



$$G_R(s) = \frac{m_0}{b_0}\frac{s + a_0}{s}$$

PI controller

Algebraic control in R_{MS} ring

Example 2: Unstable anisochronic system

$$G(s)=\frac{B(s)}{A(s)}=\frac{\frac{b(s)}{m(s)}}{\frac{a(s)}{m(s)}}=\frac{\frac{K\exp(-\tau s)}{Ts-\exp(-\vartheta s)+r_0K\exp(-\tau s)}}{\frac{Ts-\exp(-\vartheta s)}{Ts-\exp(-\vartheta s)+r_0K\exp(-\tau s)}}$$

A stable common denominator $m(s)$ according to the Michailov criterion:

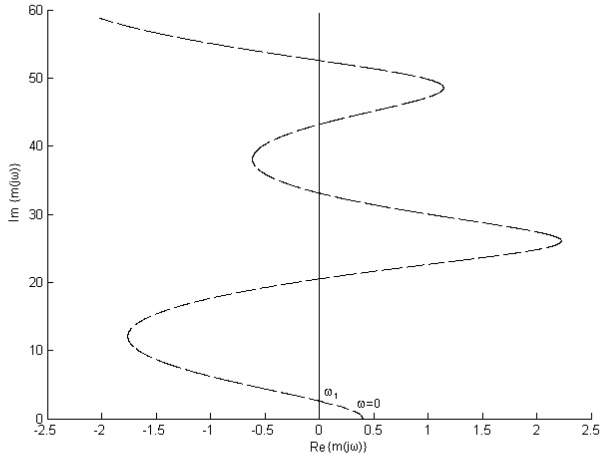
$$\lim_{\omega\to\infty}\arg\left\{m(s)\big|_{s=j\omega}\right\}=\frac{\pi}{2}$$

Quasipolynomial stability condition

$$\frac{1}{K}<r_0<\frac{1}{K}\frac{T\omega_C+\sin(\vartheta\omega_C)}{\sin(\tau\omega_C)}$$

where ω_C is the critical frequency

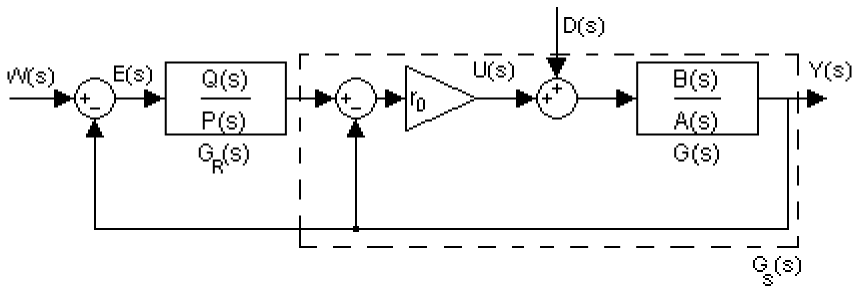
K is positive



Algebraic control in R_{MS} ring

Final controller: $$G_{R1}(s)=\frac{(r_0K+m_0T)s+m_0\left[r_0K-\exp(-\vartheta s)\right]}{K\left[s+m_0\left(1-\exp(-\tau s)\right)\right]}$$

Analogy with cascade structure



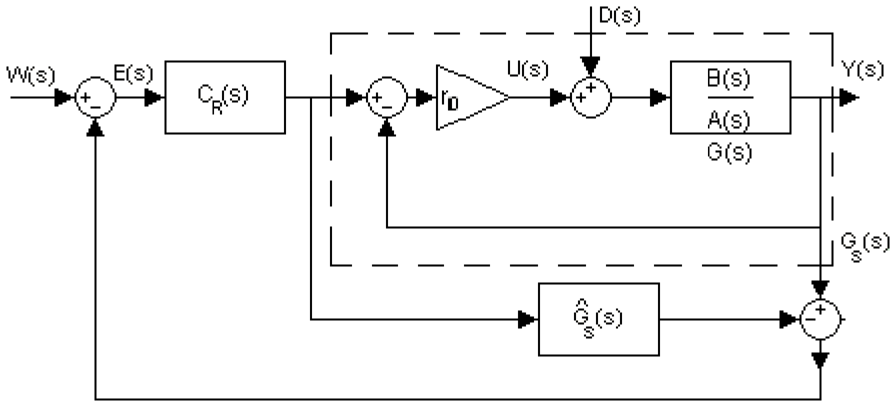
$$G_S(s)=\frac{r_0K\exp(-\tau s)}{Ts-\exp(-\vartheta s)+r_0K\exp(-\tau s)}=\frac{r_0K\exp(-\tau s)}{m(s)}$$

Final controller for the cascade structure:

$$G_{R2}(s)=\frac{m_0\left(r_0K\exp(-\tau s)+Ts-\exp(-\vartheta s)\right)}{r_0K\left[s+m_0\left(1-\exp(-\tau s)\right)\right]}$$

Algebraic control in R_{MS} ring

Comparison with IMC structure



$$G_R(s) = \frac{C_R(s)}{1 - C_R(s) G_S(s)}$$

$$G_{R3}(s) = G_{R2}(s)$$

$$= \frac{m_0 \left(r_0 K \exp(-\tau s) + Ts - \exp(-\vartheta s) \right)}{r_0 K \left[s + m_0 \left(1 - \exp(-\tau s) \right) \right]}$$

Low-pass filter was used as: $F = \dfrac{m_0}{s + m_0}$

Algebraic control in R_{MS} ring

Particular case

$$G(s) = \frac{3\exp(-4s)}{5s - \exp(-0.8s)} = \frac{\dfrac{3\exp(-4s)}{3r_0\exp(-4s) + 5s - \exp(-0.8s)}}{\dfrac{5s - \exp(-0.8s)}{3r_0\exp(-4s) + 5s - \exp(-0.8s)}}$$

Quasipolynomial stability condition: $0.333 \le r_0 \le 0.564$

Amplitude margin $A_M = 1.3 \Rightarrow r_0 = 0.434$

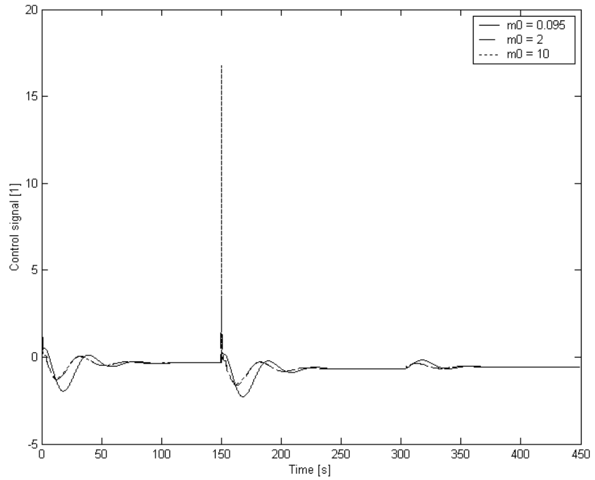
Final controllers:

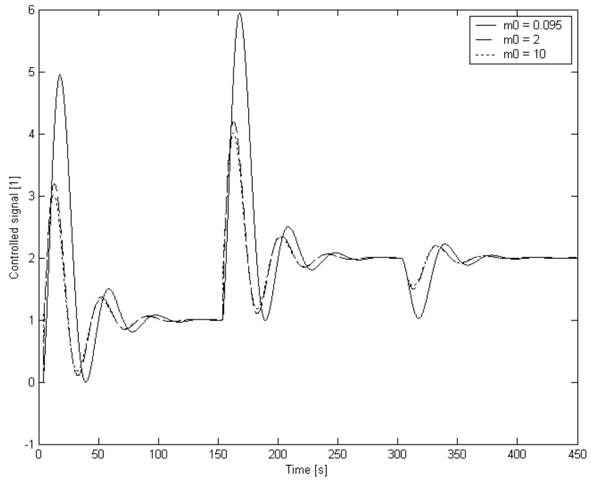
$$G_{R1}(s) = \frac{1.775s + 0.124 - 0.095\exp(-0.8s)}{3\left[s + 0.095\left(1 - \exp(-4s)\right)\right]} \qquad (m_0 = 0.095)$$

$$G_{R2}(s) = \frac{0.057\left(1.3\exp(-4s) + 5s - \exp(-0.8s)\right)}{1.3\left[s + 0.057\left(1 - \exp(-4s)\right)\right]} \qquad (m_0 = 0.057)$$

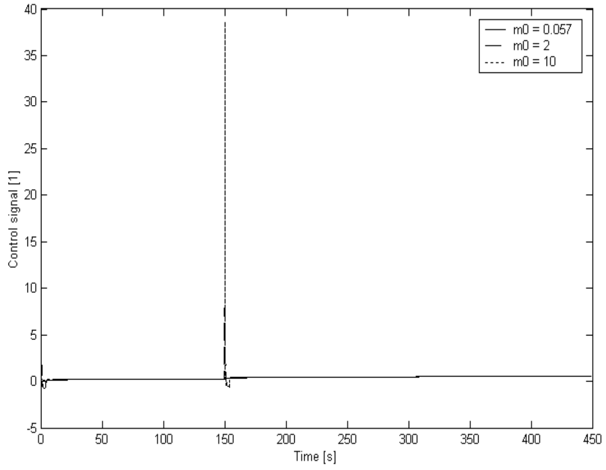
Algebraic control in R_{MS} ring

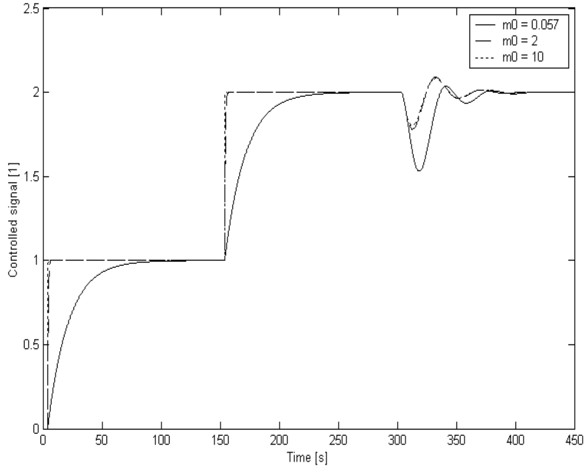
Simple 1DOF





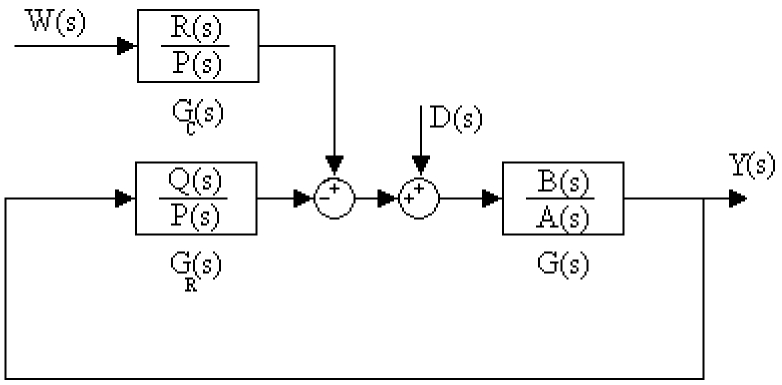
Inner loop





Algebraic control in R_{MS} ring

2DOF structure



Reference tracking:

$$\lim_{s \to 0} \left[1 - B(s)R(s) \right] = 0$$

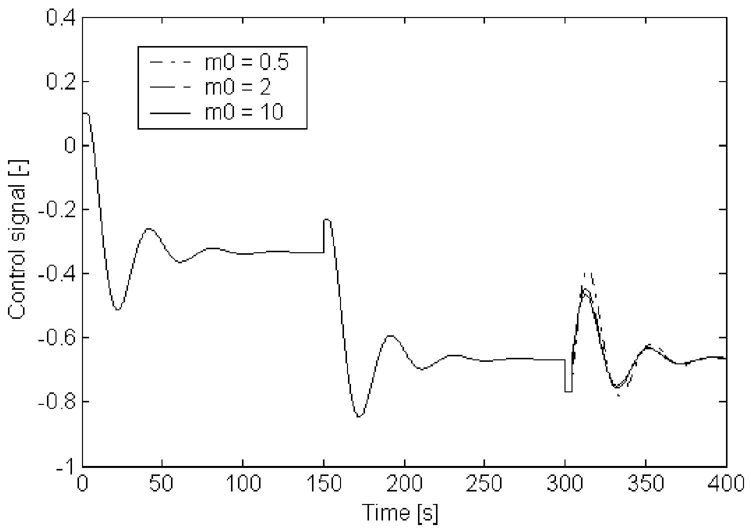
Stability and disturbance rejection as in 1DOF

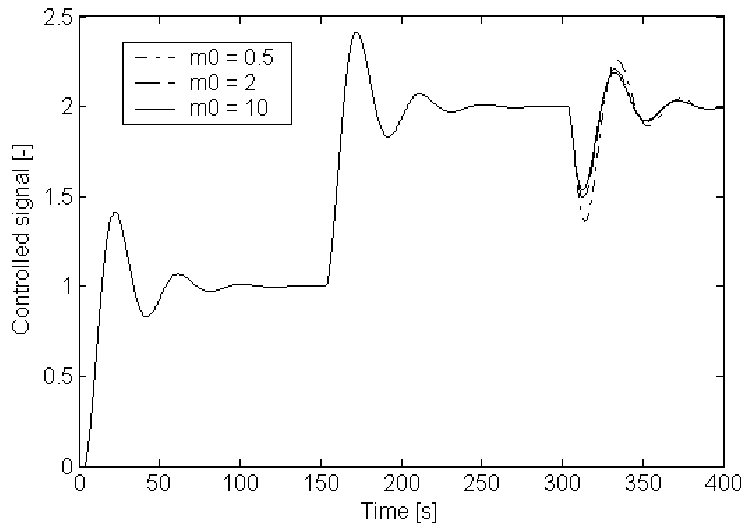
Algebraic control in R_{MS} ring

Final controllers ($r_0 = 0.434$)

$$G_R(s) = \frac{(r_0 K + T m_0)s + m_0[r_0 K - \exp(-\tau s)]}{K[s + m_0(1 - \exp(-\vartheta s))]} = \frac{(1.3 + 5m_0)s + m_0[1.3 - \exp(-0.8s)]}{3[s + m_0(1 - \exp(-4s))]}$$

$$G_C(s) = \frac{R(s)}{P(s)} = \frac{\left(r_0 - \frac{1}{K}\right)(s + m_0)}{s + m_0[\exp(-\tau s)]} = \frac{0.1(s + m_0)}{s + m_0[\exp(-4s)]}$$





Tuning methods

Equalization principle

PI controller

$$G_R(s) = K_C\left(1 + \frac{1}{T_I s}\right) = K_C + \frac{K_I}{s}$$

Requirements

$$K_C = \frac{1}{K}\frac{1 + (1-\Delta)^2}{2}, \quad T_I = (T + \tau)\frac{1 + (1-\Delta)^2}{2}, \quad \Delta = \frac{\tau}{T + \tau}$$

Simplification for anisochronic controllers (*s* -> 0)

Example:

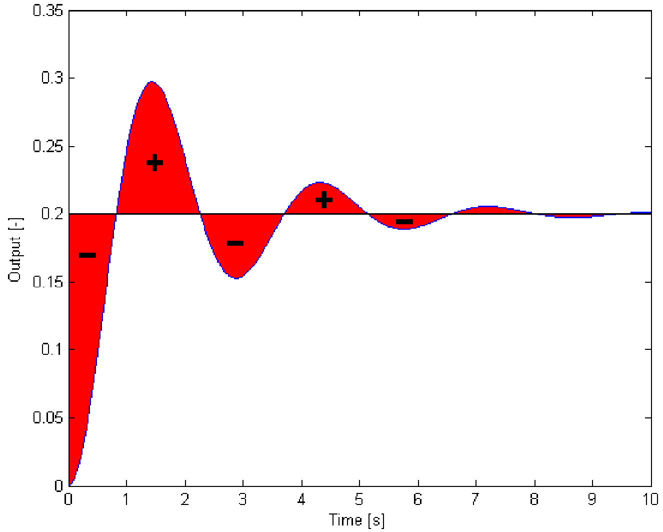
$$G_R(s) = \frac{Q(s)}{P(s)} = \frac{m_0}{K}\frac{Ts + e^{-\theta s}}{s + m_0\left(1 - e^{-\tau s}\right)} \Rightarrow \hat{G}_R(s) = \frac{m_0}{K}\frac{Ts + 1}{s}$$

Tuning methods

Integral error criterion

Minimizing the functional:

$$J_{IE} = \left| \int_0^\infty \left[y(t) - y(\infty) \right] dt \right|$$



Solution:

$$J_{IE} = \left| \lim_{s \to 0} E(s) \right| = \left| \lim_{s \to 0} \frac{\beta_{r-1} s^{r-1} + \beta_{r-2} s^{r-2} + \ldots + \beta_1 s + \beta_0}{\alpha_r s^r + \alpha_{r-1} s^{r-1} + \ldots + \alpha_1 s + \alpha_0} \right| = \left| \frac{\beta_0}{\alpha_0} \right|$$

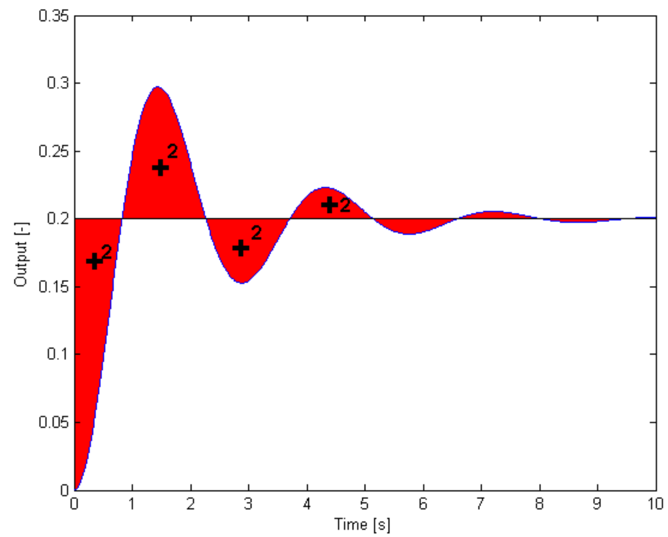
Using Lagrange multiplicators

Tuning methods

Integral squared error criterion

Minimizing the functional:

$$J_{ISE} = \int_0^{\infty} [y(t) - y(\infty)]^2 \, dt$$



Solution:

$$J_{ISE} = \frac{1}{2\alpha_r} \frac{H_1}{H}$$

where H is Hurwitz matrix of den($E(s)$)

The first row in H_1 is:

$$c_1 = (-1)^0 \beta_{r-1}^2$$

$$c_2 = (-1)^1 \left[\beta_{r-2}^2 - 2\beta_{r-1}\beta_{r-3}\right]$$

$$c_3 = (-1)^2 \left[\beta_{r-3}^2 - 2\beta_{r-2}\beta_{r-4} + 2\beta_{r-1}\beta_{r-5}\right]$$

...

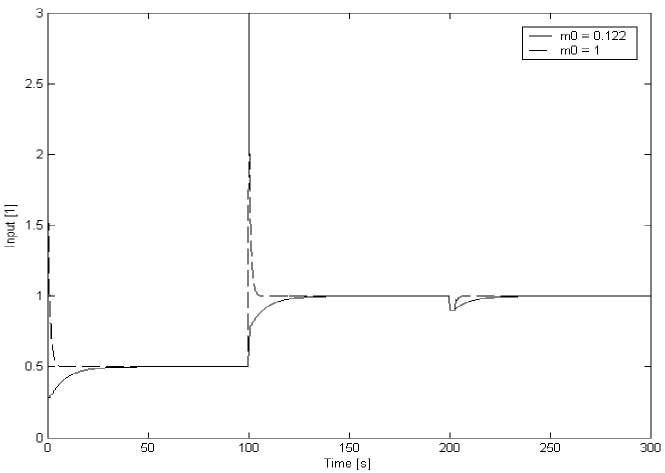
$$c_{r-1} = (-1)^{r-2} \left[\beta_1^2 - 2\beta_0\beta_2\right]$$

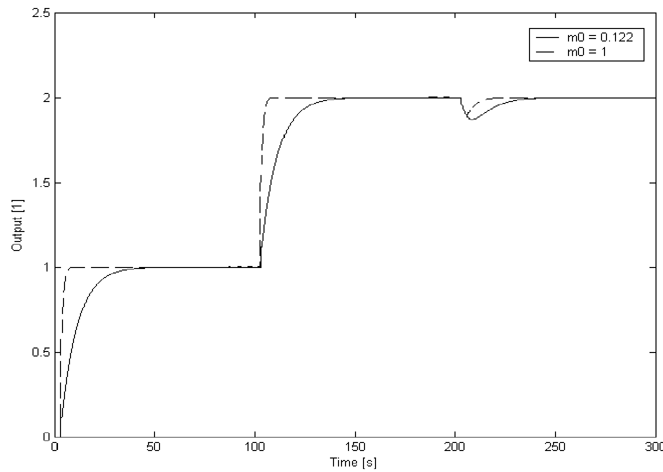
$$c_r = (-1)^{r-1} \beta_0^2$$

Tuning methods

Simple examples – algebraic design in R_{MS}

$$G(s)=\frac{B(s)}{A(s)}=\frac{\dfrac{b(s)}{m(s)}}{\dfrac{a(s)}{m(s)}}=\frac{\dfrac{2\left(s+\exp(-0.5s)\right)\exp(-3s)}{s+m_0}}{\dfrac{3s+\exp(-0.8s)}{s+m_0}}$$





Tuning methods

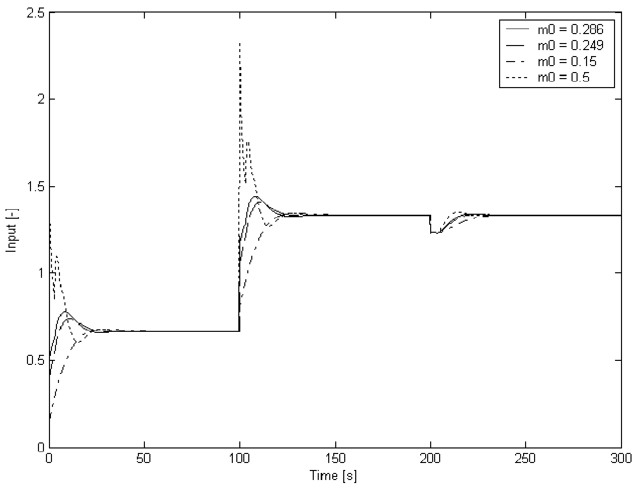
Simple examples – algebraic design in R_{MS}

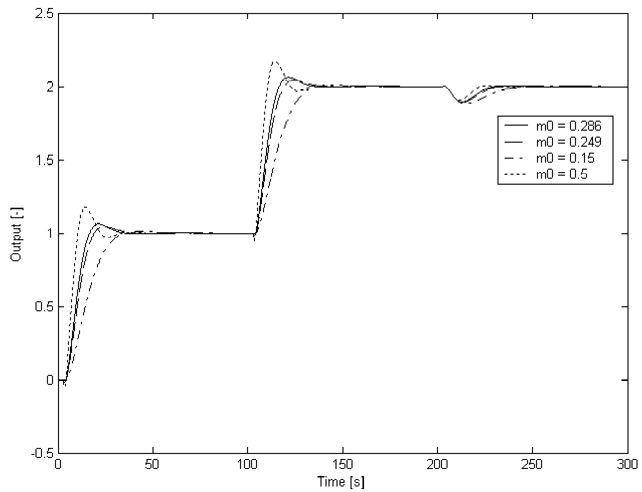
Equalization principle

$$G(s)=\frac{1.5(1-0.5s)\exp(-3s)}{(5s+\exp(-0.5s))(2s+1)}=\frac{\dfrac{1.5(1-0.5s)\exp(-3s)}{m(s)}}{\dfrac{(T_1 s+\exp(-0.5s))(2s+1)}{m(s)}}$$

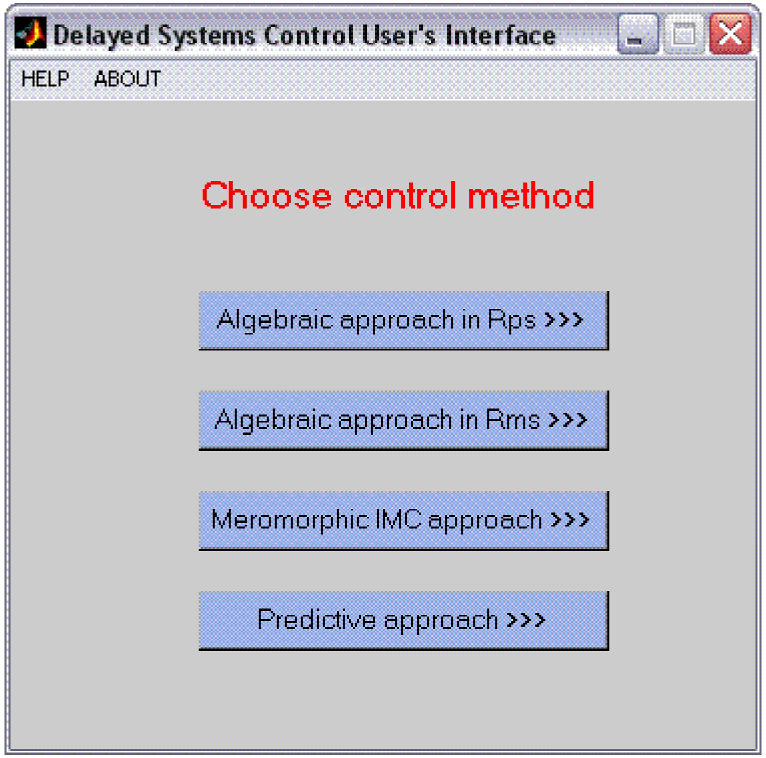
$m(s)=s^2+\sqrt{2}m_0 s+m_0^2$ (Modulus optimum)

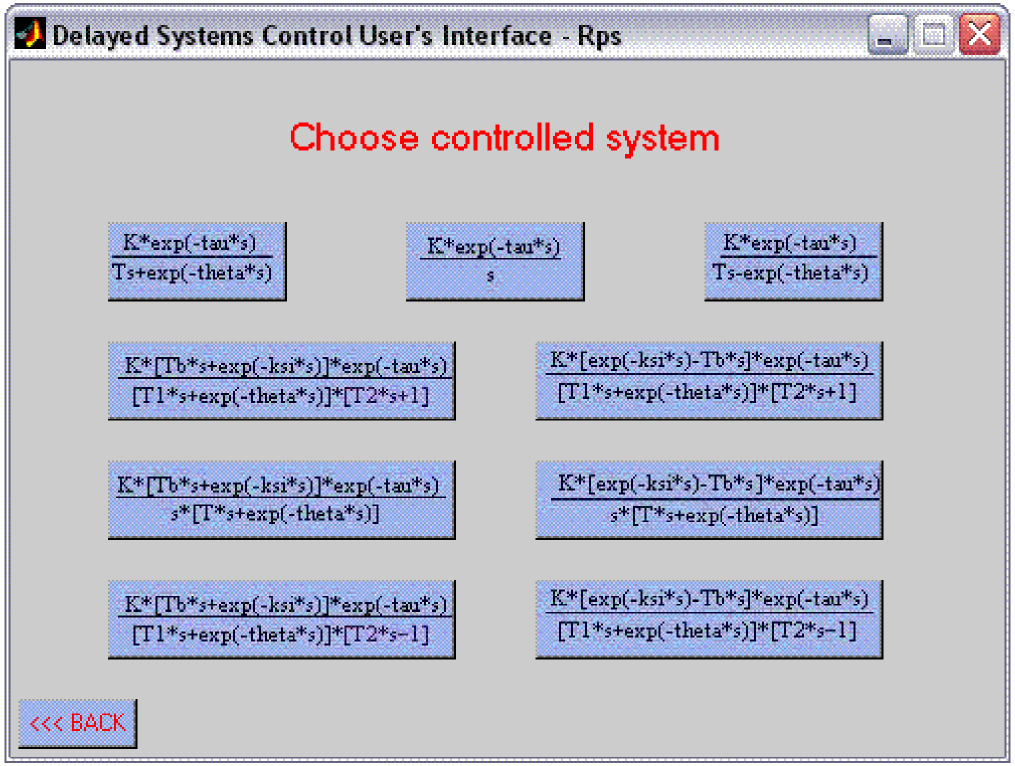
$m(s)=(s+m_0)^2$





Program implementation





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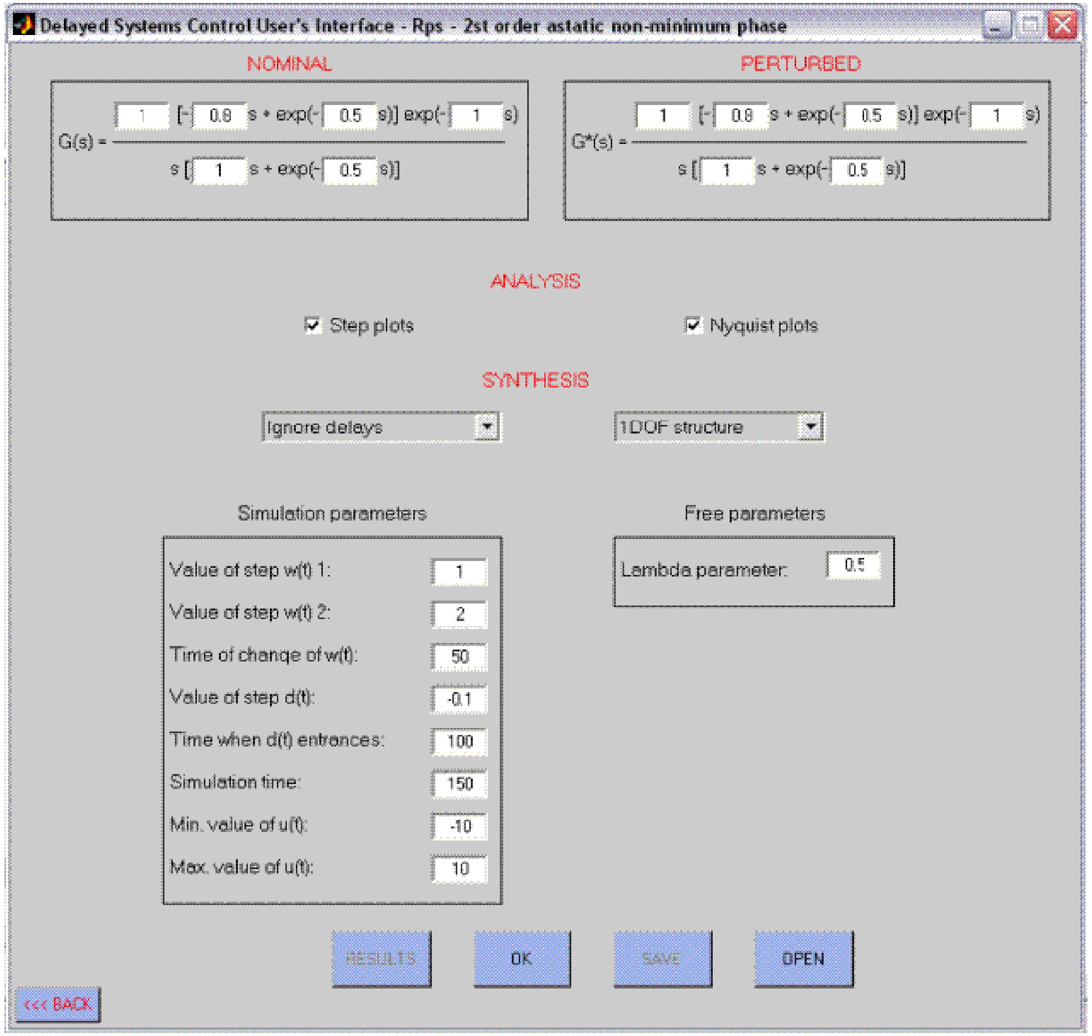

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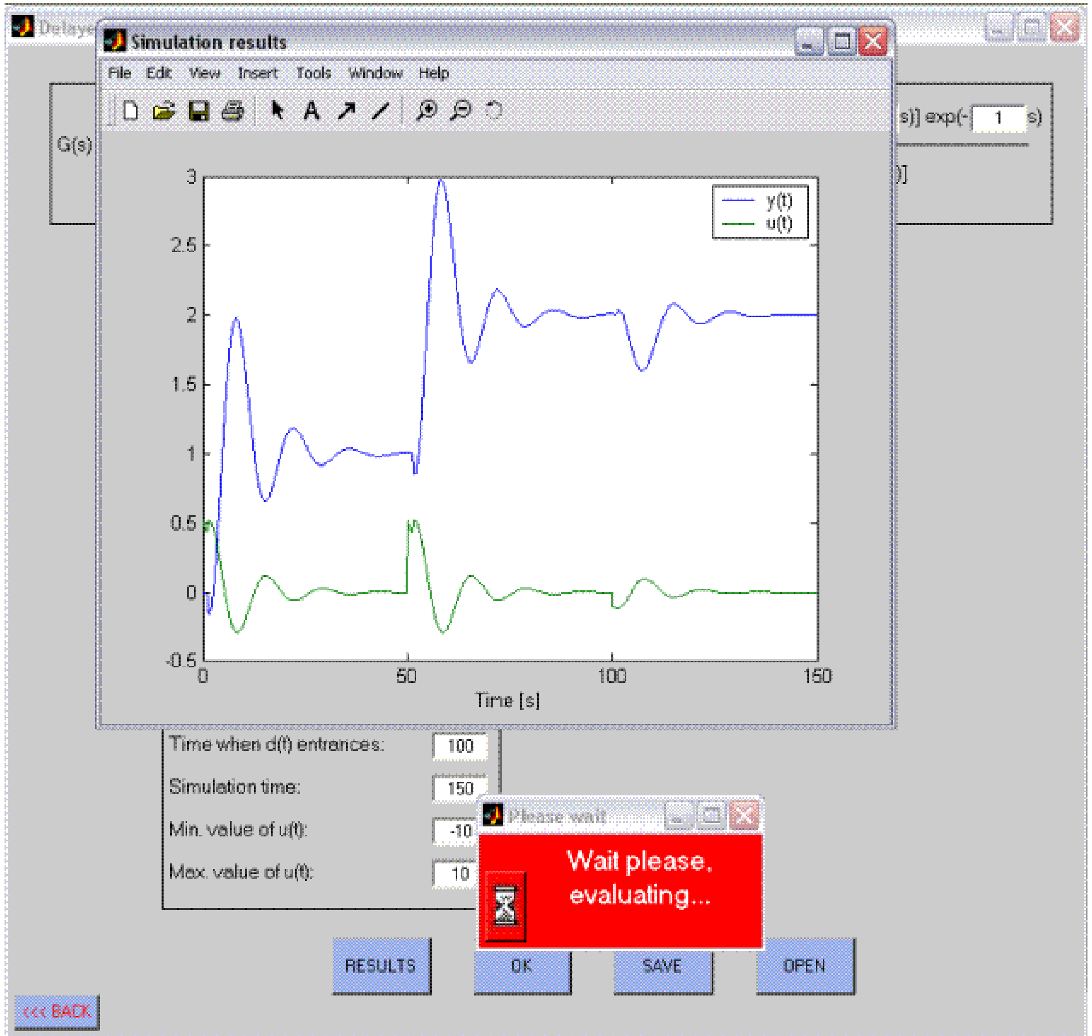
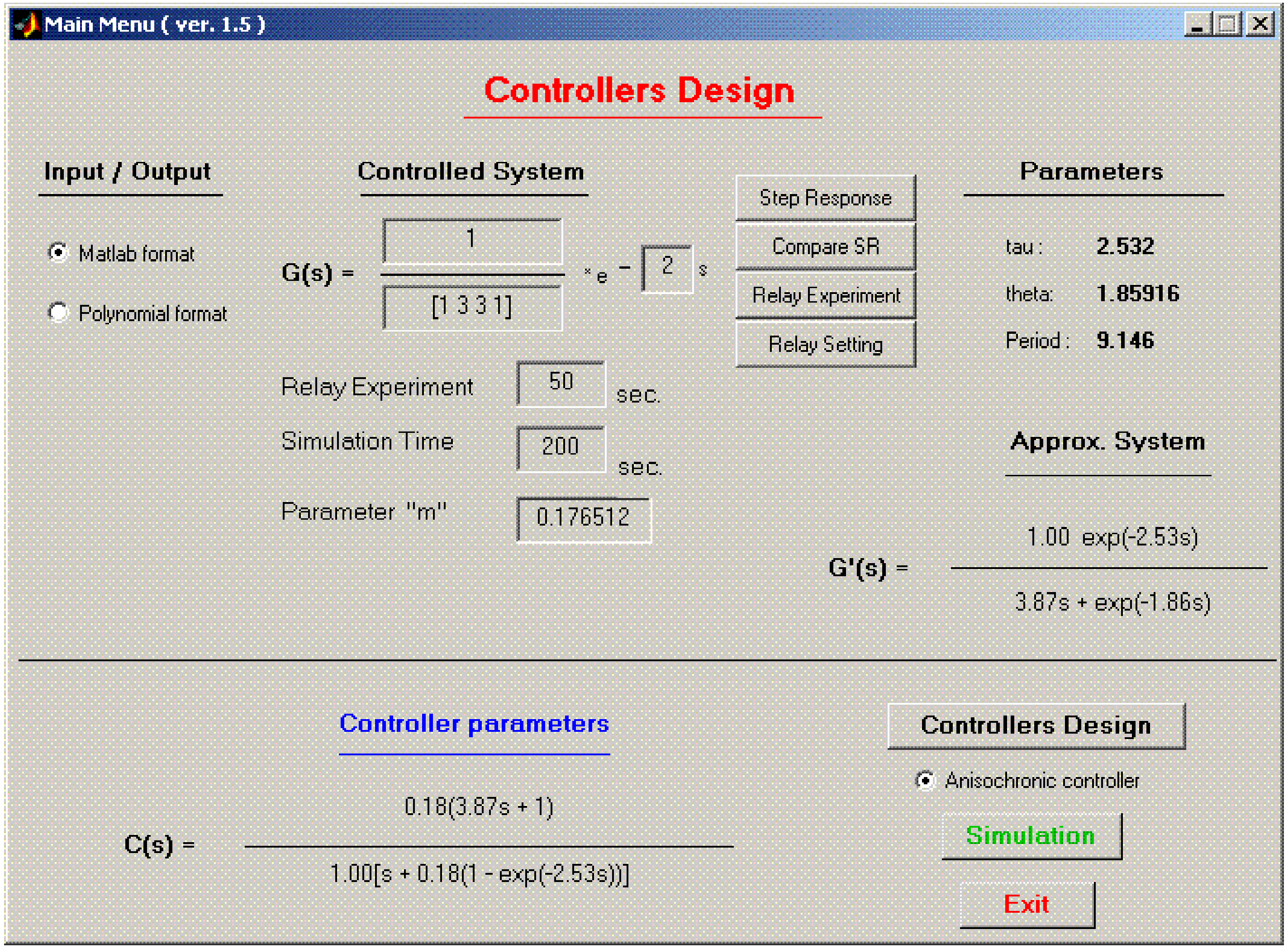
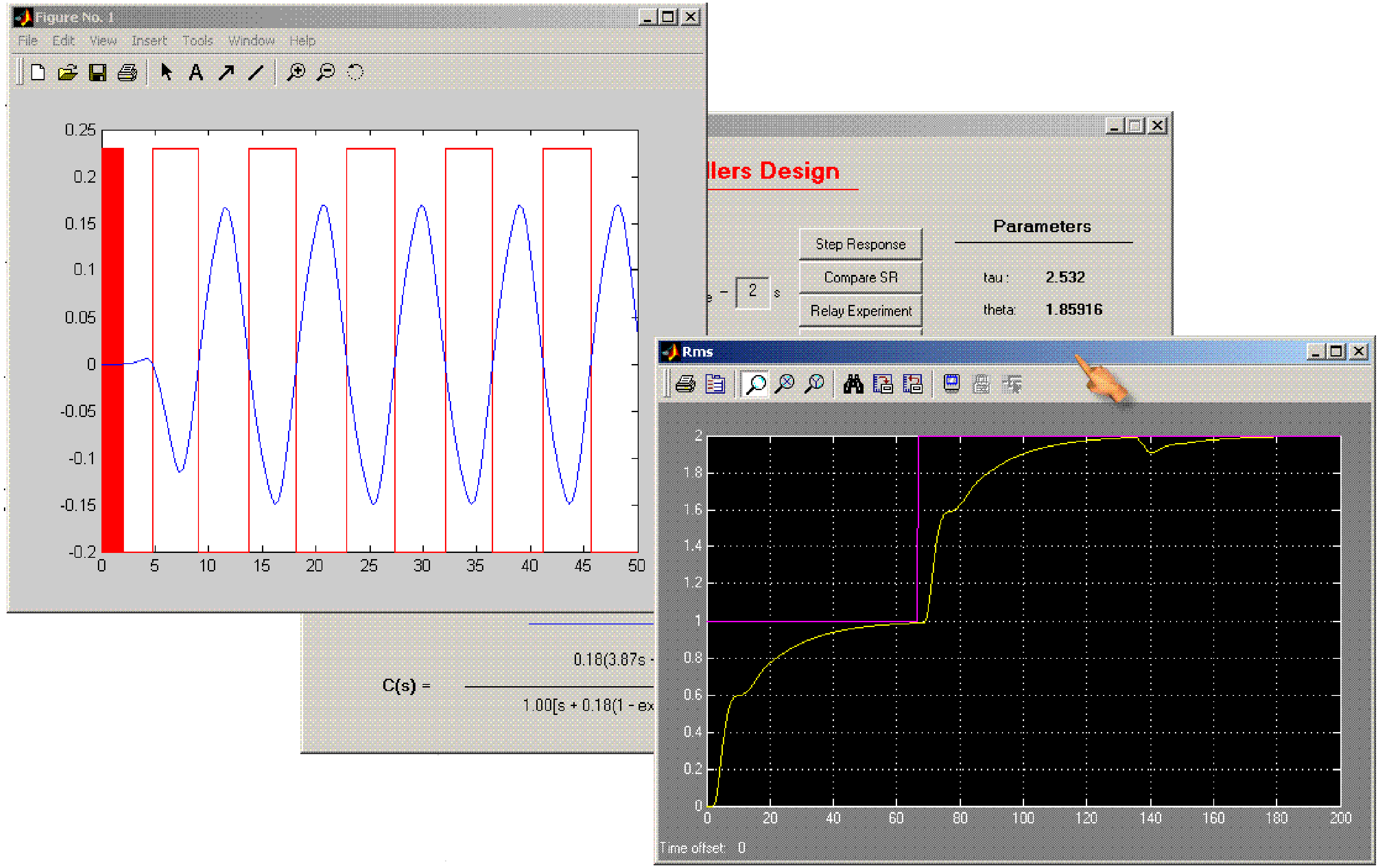


Program implementation



Program implementation

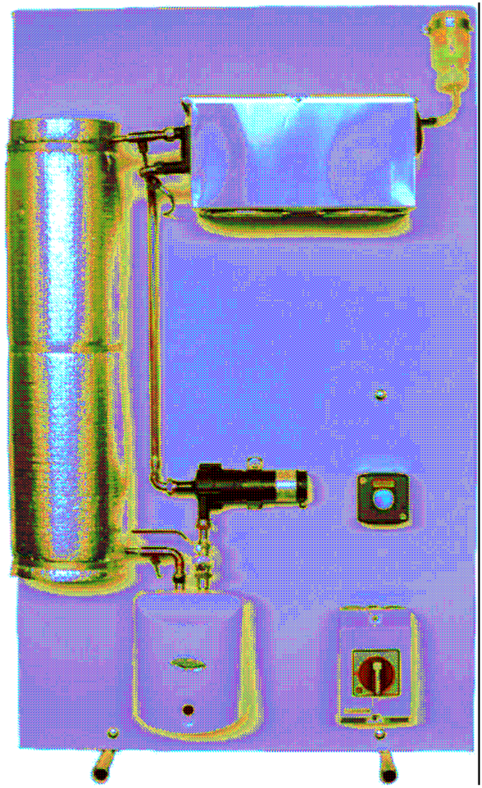


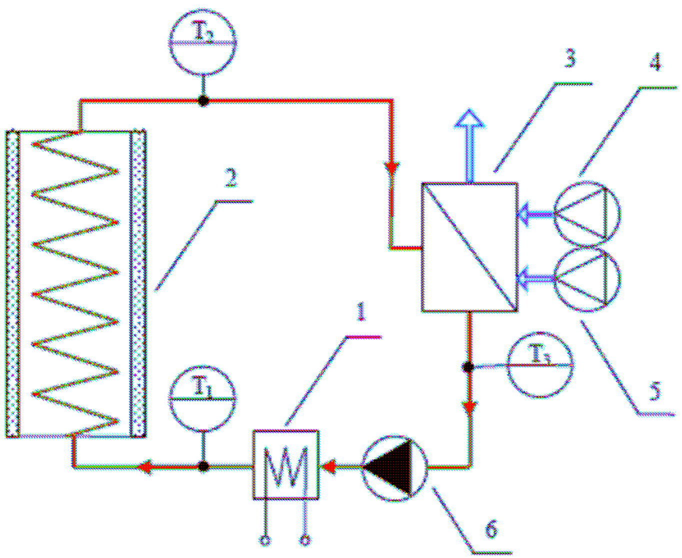
Future research

- Utilization of more complicated control systems for algebraic control approach in RMS ring
- Relay identification using other types of nonlinearities (relays)
- Investigation more sophisticated tuning methods
- Extension to MIMO systems
- Verification on laboratory model…

Future research

Laboratory model of a heat system





1. Flow heater
2. Pipeline
3. Exchanger
   (water-air)
4. Main fan
5. Secondary fan
6. Pump

Conclusions

- Limit cycle identification of anisochronic model
  - Modeling of high order systems
  - Frequency and time-domain approach
- Algebraic control in RMS ring
  - Easy controller design – Laplace transform

Thank you for your attention